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THE EFFECTS OF SOLAR RADIATION PRESSURE ON THE RADIO ASTRONOMY EXPLORER SATELLITE BOOMS

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by

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SUMMARY

The object of this paper is twofold. First to develop an efficient technique to determine the times during which the RAE satellite boom and sun are in direct line-of-sight. Second to examine the effects of solar radiation pressure alone on the Radio Astronomy Explorer satellite booms, and determine the dynamical behavior of the booms in the orbital plane.

Analytical conditions concerning the values of the eccentric anomaly E corresponding to the direct line-of-sight are obtained, and the transcendental equation giving the optimum value of the longitude of the ascending node Ω at launch is also determined. The planar motion of a Radio Astronomy Explorer satellite boom element is analyzed and the first-order solutions are expressed in terms of the radial perturbation in the orbital plane.

It is also shown that the solar radiation pressure perturbations leave the period approximately invariant.

provided

$$-\frac{\pi}{2} < \psi_1 < +\frac{\pi}{2},$$

and

$$\vec{\sigma} \cdot \vec{\eta} \geq \left[1 - \left(\frac{R}{r}\right)^2\right]^{1/2}$$

or

$$\Delta = \vec{\sigma} \cdot \vec{\eta} - \left[1 - \left(\frac{R}{r}\right)^2\right]^{1/2} \geq 0 \tag{4}$$

where

$$r = a(1 - e \cos E).$$

$\Delta > 0$ implies the visibility and $\Delta = 0$ defines the direct line-of-sight of the boom element and Sun. The parameters a, e and E are the usual orbital elements of the satellite orbit.

The components of the unit sunline vector $-\vec{\eta}$ (Sun to Earth), in terms of geocentric equatorial coordinates, can be written[5]:

$$\begin{bmatrix} \eta_x \\ \eta_y \\ \eta_z \end{bmatrix} = - \begin{bmatrix} \cos \lambda \\ \cos \epsilon \sin \lambda \\ \sin \epsilon \sin \lambda \end{bmatrix} \tag{5}$$

where λ is the longitude of the Sun and ϵ is the obliquity of the elliptic plane.

We notice that

$$\lambda = \omega_E (T_0 + t) \tag{6}$$

where ω_E is the mean orbital rate of the Earth around the Sun, T_0 is the launch time relative to the vernal equinox and t is the time after launch.

The components of the unit vector $\vec{\sigma}$ along the position vector $\vec{r}$, in terms of geocentric equatorial coordinates, are[5]:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \end{bmatrix} = \begin{bmatrix} P_1 \cos f + P_2 \sin f \\ Q_1 \cos f + Q_2 \sin f \\ R_1 \cos f + R_2 \sin f \end{bmatrix} \tag{7}$$

where

$$\left.\begin{aligned} P_1 &= \cos \omega \cos \Omega - \sin \omega \sin \Omega \cos i \\ P_2 &= - \sin \omega \cos \Omega - \cos \omega \sin \Omega \cos i \\ Q_1 &= \cos \omega \sin \Omega + \sin \omega \cos \Omega \cos i \\ Q_2 &= - \sin \omega \sin \Omega + \cos \omega \cos \Omega \cos i \\ R_1 &= \sin \omega \sin i \\ R_2 &= \cos \omega \sin i \end{aligned}\right\} \tag{8}$$

f is the true anomaly of the satellite, i is the inclination of the orbital plane, and Ω, ω are the longitude of the ascending node and argument of the perigee of the satellite respectively. We have

$$\Omega = \Omega_0 + \omega t \tag{9}$$

where Ω_0 is the longitude of the ascending node at launch and ω is the mean precession rate of the line of nodes.

Substituting (5), (7) into (4) and taking into account of the expressions of P's, Q's and R's, given by (8), we obtain:

$$\begin{aligned} \Delta = {} & \cos i \cos \lambda \sin \Omega \sin (f + \omega) \\ & - \cos \epsilon \, [\cos i \cos \Omega \sin(f + \omega) - \sin \lambda \sin \Omega \cos (f + \omega)] \\ & - \sin \epsilon \sin i \sin \lambda \sin (f + \omega) - \left[1 - \left(\frac{R}{r}\right)^2\right]^{1/2} \geq 0 \end{aligned} \tag{10}$$

We notice, through the relation

$$f = \text{Arc}\sin\left\{\frac{[a(1-e^2)]^{1/2}\sin E}{r}\right\} \tag{11}$$

that Δ is a function of the eccentric anomaly E. Therefore, one would have to solve the transcendental equation $\Delta(E) = 0$ to obtain the values of E corresponding to the direct line-of-sights.

The transcendental equation (10) gives also the optimum value of Ω_0. In fact, substituting into (10) the expressions of λ and Ω, given by (6), (9) and using $t = 0$ to indicate launch conditions, the optimum value of Ω_0, as the solution of (10) on a given launch date T_0, is obtained. One of the simplest cases would be the case of a circular orbit where the obliquity of the ecliptic is neglected and the sunline is in the equatorial plane.

Equations of Motion

The mathematical model considered is two-dimensional and the nominal circular orbit nearly equatorial and the Sun is taken to be fixed in the orbital plane. In general, orbits of arbitrary inclination contain the Sun in the orbital plane at certain times. In this study, perturbations other than those due to solar radiation pressure are not included. First order solutions will be expressed in terms of the radial perturbation.

The magnitude of the perturbing acceleration on an element of mass m and cross-section area A of the boom may be written

$$S = \frac{(1+\rho)P}{c}\left(\frac{A}{m}\right)\frac{\text{cm}}{\text{sec}^2} \tag{12}$$

where ρ is the reflectivity ($0 \le \rho \le 1$), P is the solar power constant ($P \sim 1.35 \times 10^6$ ergs/cm^2 sec), and c is the velocity of light ($c \sim 3.10^{10}$ cm/sec). The values of S are small[4] and lie in the region $10^{-4} g \le S \le 10^{-6} g$.

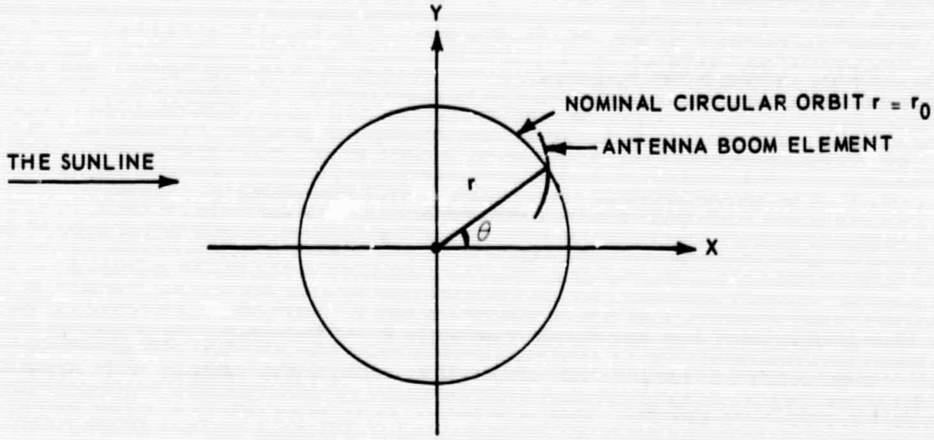

Figure 2

The components of the perturbing acceleration are

$$f_r = S\cos\theta,\ f_\theta = -S\sin\theta \tag{13}$$

and the equations of plane motion of the element in polar coordinates are:

$$\left.\begin{aligned} \ddot{r} - r\dot{\theta}^2 + \frac{\mu}{r^2} &= S\cos\theta \\ \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta}) &= -S\sin\theta \end{aligned}\right\} \tag{14}$$

subject to the initial circular orbit conditions for $t = t_0$:

$$(r)_{t_0} = r_0,\ (\theta)_{t_0} = \theta_0\ \ (\dot{r})_{t_0} = \dot{r}_0 = 0,$$
$$(r^2\dot{\theta})_{t_0} = \mu\, r_0. \tag{15}$$

the dot indicates derivation with respect to time.

The energy and angular momentum no longer constitute constants of motion because of the presence of the non-conservative force introduced by radiation pressure. As time t (or θ) increases, the angular momentum decreases.

Setting

$$G = r^2 \dot{\theta} \tag{16}$$

we have

$$\frac{dG^2}{d\theta} = -2 r^3 S \sin\theta \tag{17}$$

and

$$\begin{aligned} G^2 &= 2 S r^3 \cos\theta + C^2 \\ &= 2 S r_0^3 \cos\theta + \mu r_0, \quad G_0^2 = C^2 \end{aligned} \tag{18}$$

$$G^2 = \mu r_0 \left[1 + \frac{2 S r_0^2}{\mu} \cos\theta \right] \tag{19}$$

on the other hand,

$$\dot{r} = \frac{dr}{dt} = \frac{dr}{d\theta} \cdot \frac{d\theta}{dt} = \dot{\theta} \frac{dr}{d\theta} = -\frac{d}{d\theta}\left(\frac{G}{r}\right),$$

and

$$\ddot{r} = -\frac{d^2 r}{d\theta^2}\left(\frac{G}{r}\right) \cdot \frac{G}{r}$$

Changing the independent variable time t to θ and the familiar change of variable $u = 1/r$, the first equation of (13) becomes:

$$\frac{d^2 u}{d\theta^2} + u = \frac{\mu}{G^2} - \frac{1}{G^2} r^3 S \sin\theta \frac{du}{d\theta} - \frac{S\cos\theta}{u^2 G^2} \tag{20}$$

But

$$\frac{1}{G^2} = \frac{1}{\mu\, r_0}\left(1 - \frac{2\, S\, r_0^2 \cos\theta}{\mu}\right),$$

and if the initial values

$$u = \frac{1}{r_0}, \quad \frac{du}{d\theta} = 0,$$

are used, equation (20) will reduce to

$$\frac{d^2u}{d\theta^2} + u = \frac{1}{r_0} - \frac{3\, S\, r_0 \cos\theta}{\mu} \tag{21}$$

the solution satisfying the initial conditions $\theta_0 = 0$ is

$$u = \frac{1}{r} = \frac{1}{r_0} - \frac{3}{2}\,\frac{S\, r_0}{\mu}\,\theta \sin\theta. \tag{22}$$

The above solution is physically not acceptable, and the solution of (21) satisfying a more convenient initial condition $\theta_0 = -\pi/2$ is

$$u = \frac{1}{r} = \frac{1}{r_0} - \frac{3}{2}\,\frac{S\, r_0}{\mu}\left[\cos\theta + \left(\frac{\pi}{2} + \theta\right)\sin\theta\right]$$

or retaining only the leading terms in S:

$$r \simeq r_0 + \frac{3}{2}\,\frac{S\, r_0^3}{\mu}\left[\cos\theta + \left(\frac{\pi}{2} + \theta\right)\sin\theta\right] \tag{23}$$

the trajectory of the antenna boom element is no longer a circle and the perturbation is given by the second term of the right-hand side of (23).

All terms of (23) are bounded and periodic except the secular term $\theta \sin\theta$, which after a number of revolutions will be the dominant term. The perigee and apogee points are defined by the condition that $dr/d\theta = 0$ at both of them. Therefore the angular positions of perigee and apogee are given by

$$\frac{dr}{d\theta} = \frac{3}{2} \frac{S r_0^3}{\mu} \left(\frac{\pi}{2} + \theta \right) \cos\theta = 0 \tag{24}$$

or

$$\left(\frac{\pi}{2} + \theta \right) \cos\theta = 0 \tag{25}$$

therefore

$$\theta_{perigee} = 2K\pi - \frac{\pi}{2}, \quad \theta_{apogee} = 2K\pi + \frac{\pi}{2} \tag{26}$$

where $K = 0, 1, \ldots$ is the number of revolutions.

The perigee and apogee radii are given by

$$\left.\begin{aligned} r_{perigee} &= r_0 \left[1 - \frac{3}{2} \frac{S r_0^2}{\mu} 2K\pi \right] \\ r_{apogee} &= r_0 \left[1 + \frac{3}{2} \frac{S r_0^2}{\mu} (2K\pi + \pi) \right]. \end{aligned}\right\} \tag{27}$$

The semimajor axis a given by:

$$a = \frac{1}{2} (r_{perigee} + r_{apogee}) = r_0 \left(1 + \frac{3}{4} \frac{S r_0^2}{\mu} \pi \right)$$

is constant and then the period of revolution

$$T = 2\pi \left(\frac{a^3}{\mu} \right)^{1/2} \tag{28}$$

is approximately constant. Therefore, the solar radiation pressure perturbations leave period approximately invariant.

The first formulae of (26) and (27) lead us to believe that the altitude of the perigee decreases linearly with the number of revolutions K and the magnitude of the perturbing acceleration S, so that the altitude of the perigee tends to zero after K_1 revolutions, where

$$K_1 = \mu(3\, S\, r_0^2\, \pi)^{-1} \text{ (nearest integer)} \tag{29}$$

Figure 3 shows the variation of the perigee's altitude with respect to K for a given value of S. Relation (29) shows that the ideal number of revolutions for which the perigee's altitude tends to zero is inversely proportional to the magnitude of perturbing acceleration.

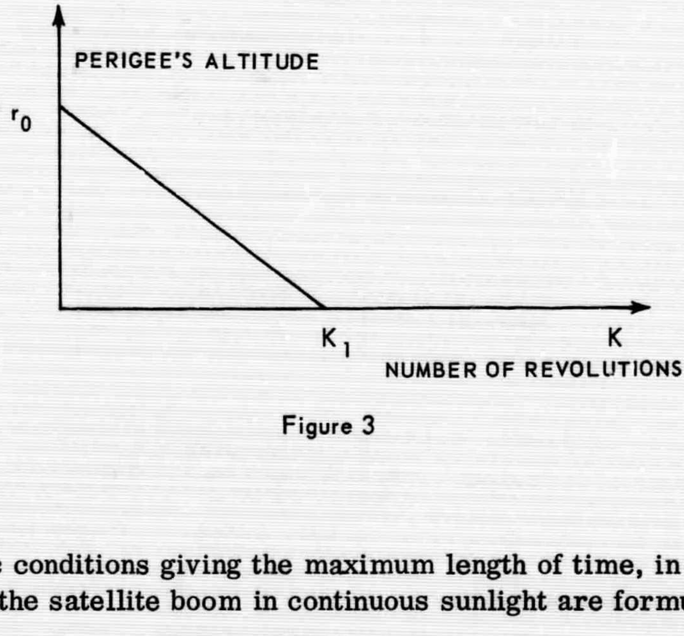

Figure 3

CONCLUSIONS

The analytic conditions giving the maximum length of time, in terms of launch date, for the satellite boom in continuous sunlight are formulated.

A simple two-dimensional model is considered and the Sun is taken to be in the orbital plane. Perturbations other than those due to solar radiation pressure are not included. It is found that for this simple two-dimensional model of fixed coplanar Sun the perigee's altitude decreases linearly with the number of revolutions and the magnitude of perturbing acceleration with an almost constant period.

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